Warmups

① Simplify

$(A+B)^2 = A^2 + 2AB + B^2$

ⓐ $\dfrac{x^5(x^2+x^3)^2}{x^3\sqrt{x}}$

$\cancel{= \dfrac{x^5(x^4+x^6)}{x^3\sqrt{x}}}$ $= \dfrac{x^5(x^4+2\cdot x^2\cdot x^3+x^6)}{x^3\sqrt{x}}$

$= \dfrac{x^5(x^4+2x^5+x^6)}{x^3\sqrt{x}} = \dfrac{x^9+2x^{10}+x^{11}}{x^3\sqrt{x}}$

$= \dfrac{x^9}{x^3\sqrt{x}} + \dfrac{2x^{10}}{x^3\sqrt{x}} + \dfrac{x^{11}}{x^3\sqrt{x}} = \dfrac{x^6}{\sqrt{x}} + \dfrac{2x^7}{\sqrt{x}} + \dfrac{x^8}{\sqrt{x}}$

$= \dfrac{x^6}{x^{1/2}} + \dfrac{2x^7}{x^{1/2}} + \dfrac{x^8}{x^{1/2}} = \boxed{x^{11/2} + 2x^{13/2} + x^{15/2}}$

OR: $\dfrac{x^5(x^2+x^3)^2}{x^3\sqrt{x}} = \dfrac{x^5(x^2(1+x))^2}{x^3x^{1/2}}$

$= \dfrac{x^5x^4(1+x)^2}{x^{7/2}} = \dfrac{x^9(1+x)^2}{x^{7/2}} = \boxed{x^{11/2}(1+x)^2}$

(b) $\dfrac{\frac{a+4}{a-2}+2}{a} = \dfrac{\frac{a+4}{a-2}+\frac{2(a-2)}{(a-2)}}{a}$

$= \dfrac{\frac{a+4+2(a-2)}{a-2}}{a} = \dfrac{\frac{a+4+2a-4}{a-2}}{a}$

$= \dfrac{\frac{3a}{a-2}}{a/1} = \dfrac{3a}{(a-2)}\cdot\dfrac{1}{a} = \dfrac{3\cancel{a}}{(a-2)\cancel{a}}$

$= \boxed{\dfrac{3}{a-2}}$

(c) $\sqrt{\dfrac{(2b+1)^3-(1-b)}{b^5}} =$

Pascal's Triangle:

1
1 1
1 2 1
→ 1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$(A+B)^3 = A^3+3A^2B+3AB^2+B^3$

$(2b+1)^3 = (2b)^3+3(2b)^2\cdot 1+3(2b)\cdot 1^2+1^3$

$= 8b^3+3\cdot 4b^2+3\cdot 2b+1$

$= 8b^3+12b^2+6b+1$

$$= \sqrt{\frac{(8b^3+12b^2+6b+1)-(1-b)}{b^5}}$$

$$= \sqrt{\frac{8b^3+12b^2+6b+1-1+b}{b^5}}$$

$$= \sqrt{\frac{8b^3+12b^2+7b}{b^5}}$$

$$= \sqrt{\frac{\cancel{b}(8b^2+12b+7)}{\cancel{b^5}\, b^4}}$$

$$= \frac{\sqrt{8b^2+12b+7}}{\sqrt{b^4}} = \boxed{\frac{\sqrt{8b^2+12b+7}}{b^2}}$$

d) $\tan(\underbrace{\arctan(p)}_{\text{angle whose tangent is } p}) = \boxed{p}$